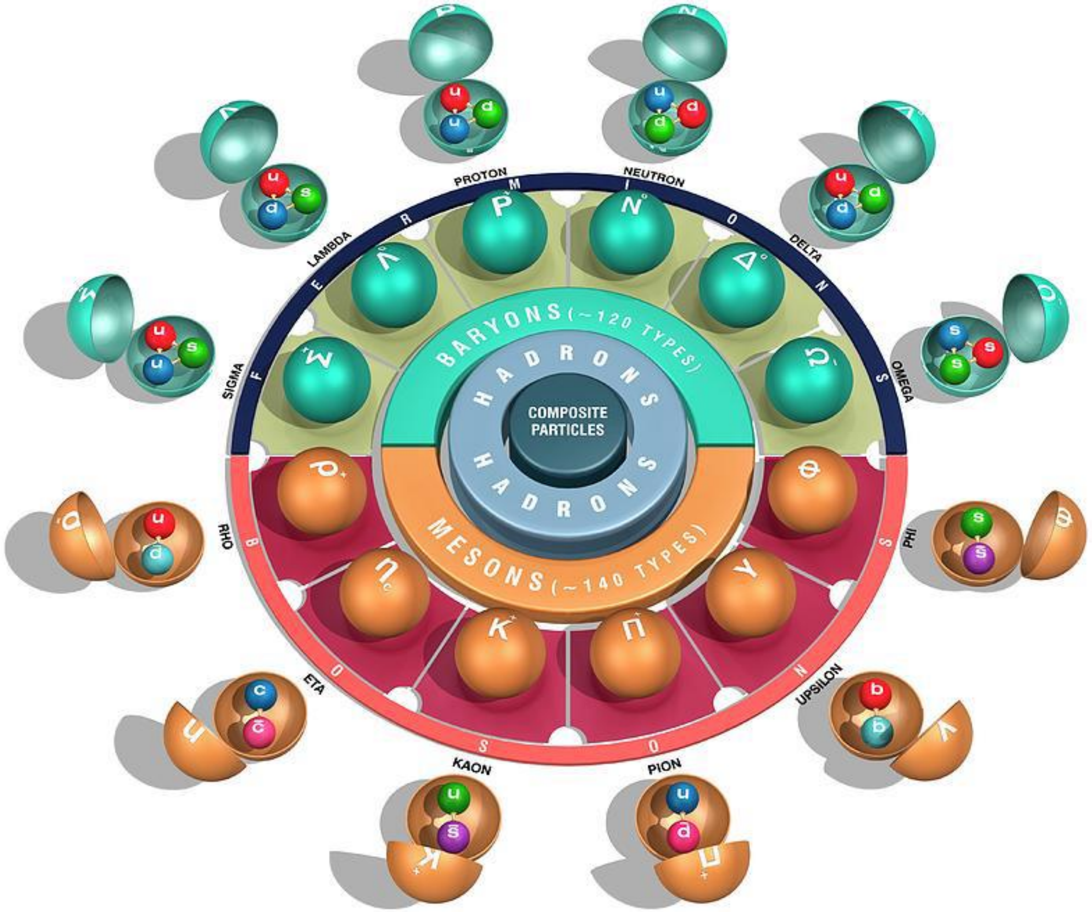

Particle Physics I

Lecture 13: Discrete symmetries – C, P, T

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December 10th 2025

Learning targets

Learning targets

- Discussion on discrete symmetries: charge conjugation and parity
- Symmetry transformation for fermions
- Meson parity, helicity and chirality
- Examples of symmetry violation

Symmetries

- The symmetries of empty space should be respected by a physical theory
  - example: there is no preferred direction in space $\Longrightarrow$ any experiment should give the same results before and after rotation
- In addition to the continuous space-time symmetries (rotations, translations) there are two discrete transformations
  - Parity (P): $(t, \vec{x}) \rightarrow (t, -\vec{x})$
  - Time reversal (T): $(t, \vec{x}) \rightarrow (-t, \vec{x})$
- From the definition, it follows that $P^2 = T^2 = 1$

Symmetries: parity

- We say that a theory obeys *P* symmetry if there is no experiment that can distinguish between a world and its mirror image

P-even	**P-odd**
?	?

Symmetries: parity

- We say that a theory obeys P symmetry if the is no experiment that can distinguish between a world and its mirror image

P-even	**P-odd**
time t	position $\vec{x}$
angular momentum	momentum $\vec{p}$
mass density	force
electric charge	electric current
magnetic field	electric field

Symmetries: parity

- Parity transformation on a scalar field φ and a vector field A_μ

$$\varphi(\vec{x},t) \rightarrow \varphi'(\vec{x},t) = \varphi(-\vec{x},t)$$

$$\begin{pmatrix} A_0(\vec{x},t) \\ \vec{A}(\vec{x},t) \end{pmatrix} \rightarrow \begin{pmatrix} A'_0(\vec{x},t) \\ \vec{A}'(\vec{x},t) \end{pmatrix} = \begin{pmatrix} A_0(-\vec{x},t) \\ -\vec{A}(-\vec{x},t) \end{pmatrix}$$

- For a plane wave, a coordinate transformation defines how the momentum transforms

$$\phi_{\vec{p}} \propto e^{i\vec{p}\cdot\vec{x}} \rightarrow e^{-i\vec{p}\cdot\vec{x}} \propto \phi_{-\vec{p}} \qquad \vec{p} \rightarrow -\vec{p}$$

- For electromagnetic field, polarization vector reflects as well

$$\vec{A}_{\vec{p}} \propto \epsilon e^{i\vec{p}\cdot\vec{x}} \rightarrow -\epsilon e^{-i\vec{p}\cdot\vec{x}} \propto -\vec{A}_{-\vec{p}} \qquad \epsilon \rightarrow -\epsilon$$

What about fermions?

Transform non-trivially under rotations and Lorentz transformations, but not as normal vectors.
What do they see in the mirror?

Fermion quantum numbers

- Let's start by first fixing the electron state using quantum numbers – eigenvalues of operators that commute with the Hamiltonian
- For the Dirac Hamiltonian there are two such quantities: momentum and helicity!

Momentum: $\left[\hat{\vec{p}}, H\right] = [-i\nabla, H] = 0$

Helicity $\left(h \equiv \frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}\right)$: $\left[\vec{\Sigma}\cdot\vec{p}, H\right] = [-i\nabla, H] = 0$ (lecture/exercise sheet 6)

- The two operators are also commuting with each other: $[p, h] = 0$
- The state of a fermion can be described by its *energy* E, *momentum* p, and *helicity* h
- States with fixed $(E, \vec{p}, h)$ form a basis in Hilbert space of Dirac fermions

Parity transformation for fermions

- The parity transformation acts on the basis states as $(E, \vec{p}, h) \rightarrow (E, -\vec{p}, -h)$
- How does it act on an arbitrary state ψ?
- Dirac equation transforms as:

$$\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi(\vec{x}, t) = \left(i\gamma^{0}\partial_{0} + i\gamma^{i}\partial_{i} - m\right)\psi \rightarrow \left(i\left(\gamma^{\mu}\partial_{\mu}\right)' - m\right)\psi_{P} = \left(i\gamma^{0}\partial_{0} - i\gamma^{i}\partial_{i} - m\right)\psi_{P}$$

- Where ψ_P is a spinor ψ after parity transformation
- To restore the initial Dirac equation, one should take $\psi_P = \gamma^0\psi(-\vec{x}, t)$ so that

$$P\psi(\vec{x}, t) = \gamma^{0}\psi(-\vec{x}, t)$$

- Example: show explicitly that the helicity is a P −odd quantity $(Ph = -hP)$

Chirality

- In the given reference frame, the basis of Dirac fermions can be chosen as $(E, \vec{p}, h)$
- However: helicity is not Lorentz-invariant as long as the mass of the particle is not zero
- For any particle with $m \neq 0$ we can always make a Lorentz boost flipping the direction of $\vec{p}$ and therefore flipping the sign of h: $h \rightarrow -h$
- There exists a similar P −odd scalar quantity, that respects relativity but in general is not conserved: **chirality**
- In the massless limit helicity and chirality coincide meaning that all the states with definite h also have definite chirality

Chirality

$$\psi = \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix}$$

⇧

- In the Weyl basis $\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \tilde{\sigma}^\mu & 0 \end{pmatrix}$ the spinor ψ can be split into two independent parts Ψ_L, Ψ_R
- These parts evolve independently for massless particles:

$$(i\gamma^\mu \partial_\mu)\psi = \begin{pmatrix} 0 & i(\partial_t + \vec{\sigma}\cdot\vec{\nabla}) \\ i(\partial_t - \vec{\sigma}\cdot\vec{\nabla}) & 0 \end{pmatrix} \begin{pmatrix} \Psi_L \\ \Psi_R \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \Longrightarrow \quad \begin{aligned} i\left(\partial_t + \vec{\sigma}\cdot\vec{\nabla}\right)\Psi_\mathrm{R} &= 0 \\ i\left(\partial_t - \vec{\sigma}\cdot\vec{\nabla}\right)\Psi_L &= 0 \end{aligned}$$

$$\psi_L = \begin{pmatrix} \Psi_L \\ 0 \end{pmatrix}$$

Two independent free particles

$$\psi_R = \begin{pmatrix} 0 \\ \Psi_R \end{pmatrix}$$

- We call ψ_L left-chiral spinor and ψ_R right-chiral spinor

Chirality

- Left and right-chiral spinors $\psi_{L/R}$ are eigenstates of the *chirality operator* γ^5

$$\gamma_5 = i\gamma^0\gamma^1\gamma^2\gamma^3 = \begin{pmatrix} -\mathbb{I} & 0 \\ 0 & \mathbb{I} \end{pmatrix} \Longrightarrow$$

$$\gamma_5 \Psi_\mathrm{R} = \Psi_\mathrm{R}$$

$$\gamma_5 \Psi_L = -\Psi_\mathrm{L}$$

- We can extract left and right-chiral parts of the arbitrary spinor ψ using the *chirality projectors*

$$\psi_L = \frac{1-\gamma_5}{2}\psi \qquad \psi_R = \frac{1+\gamma_5}{2}\psi$$

Relation between helicity and chirality

- Chirality matrix anticommutes with all γ matrices

$$\{\gamma^{\mu}, \gamma_5\} \; = 0$$

- In particular, $\gamma_5\gamma^0 = -\gamma^0\gamma_5$. Hence *chirality* is *P-odd*, i.e. $P\psi_{L/\mathrm{R}} = \psi_{R/L}$
- Unlike helicity, chirality is Lorentz-invariant, but not conserved as long as $m \neq 0$
- Let us find relation between helicity and chirality for a massless particles
- For a plane wave ansatz $\psi_{\vec{p}}(\vec{x}) = \exp\left[-p_{\mu}x^{\mu}\right]u(\vec{p})$ Dirac equation gives: $p_0 u(\vec{p}) = \gamma_0(\vec{\gamma} \cdot \vec{p})u(\vec{p})$
  - multiplying it by γ_5 from the left and using $\gamma_5\gamma^0\gamma^i = \Sigma_i$

$$\gamma_5\psi \begin{cases} +2\hat{h}\psi, & p_0 > 0 \\ -2\hat{h}\psi, & p_0 < 0 \end{cases}$$

- i.e. for massless particle chirality and helicity are defined simultaneously (however, up to a minus sign for antiparticles)

Summary on helicity and chirality

1. Helicity is conserved and related to the momentum and spin of the particle ⟹ measurable physical quantity
2. Chirality is an unobservable mathematical construction, allowing correct relativistic description
3. Both are P −odd
4. For massless particles, states with definite helicity and chirality coincide

Meson parity

- Parity of mesons

$$P(q\bar{q}) = P(q)P(\bar{q}) \times (-1)^l = (+1)(-1)(-1)^l = (-1)^{l+1}$$

using that we defined the intrinsic parity of particles as $+1$ (and hence that of antiparticles as -1)

- As a consequence, $l = 0$ mesons have odd intrinsic parity
- The photon has parity -1, as have all other exchange particles (vector bosons)
- Parity is conserved in QED and QCD (but not in the weak interaction)

Symmetries of QED

$$\left(i\gamma^{\mu}\partial_{\mu} - e\gamma^{\mu}A_{\mu} - m\right)\psi(\vec{x},t) = 0$$

- **Parity *P***: $\psi(\vec{x},t) \to \gamma^{0}\psi(-\vec{x},t)$
  - A_{μ} transforms in the same way as ∂_{μ} - this is a symmetry of the full Hamiltonian
  - P interchanges ψ_L and ψ_R $\left(P_L\psi_{R/L} = \psi_{L/R}\right)$ *electromagnetic interaction does not distinguish two components and interacts with both of them in the same way*
- **Charge conjugation *C***: $\psi(\vec{x},t) \to -i\gamma^{2}\psi^{*}(\vec{x},t)$
  - C swaps particles for antiparticles
  - The symmetry of the total Hamiltonian of fermions and photons also involves transformation $A_{\mu} \to A_{\mu}^{C}$

$$\left(i\gamma^{\mu}\partial_{\mu} + e\gamma^{\mu}A_{\mu} - m\right)\psi^{C} \Longrightarrow \left(i\gamma^{\mu}\partial_{\mu} - e\gamma^{\mu}A_{\mu}^{C} - m\right)\psi^{C}$$

$$\boxed{A_{\mu}^{C} = -A_{\mu}}$$

- As a consequence, photons have $C = -1$ (EM field changes sign under C)

Charge conjugation eigenstates

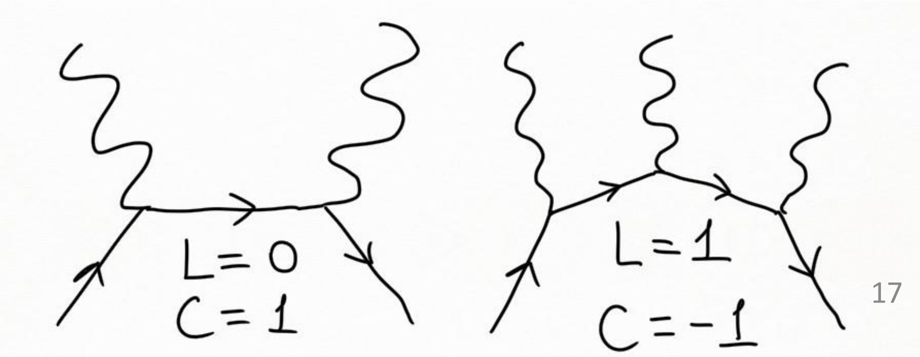
- Most particles are not eigenstates of C (e.g. leptons, quarks, charged pions) − particle needs to be its own antiparticle
- Photons, and neutral mesons that are combinations of $u\bar{u}, d\bar{d}, s\bar{s}, c\bar{c}$ **are** eigenstates
- Note that being neutral does not imply that a particle is a charge conjugation eigenstate, e.g. neutron $C|n\rangle \rightarrow |\bar{n}\rangle$
- System consisting of a fermion and its antiparticle is eigenstate with $C = (-1)^{l+s}$ (compare with parity $P = (-1)^{l+1}$)

Symmetries in QED: examples I

- Bound system of interacting electron + positron may be in different states, similar to that of hydrogen atom
- For instance, they can be in state with zero spin momentum – *parapositronium*, $S = 0$, and an exited state with $S = 1$ is called *ortopositronium*
- When one applies C −transformation to a system of particle + antiparticle, the wave function $\psi(e^-, e^+)$ turns to $\psi(e^+, e^-)$, which differs from the initial one by a factor of $(-1)^J$ due to permutation of fermions
- Spatial permutation of two fermions gives -1 times parity factor $(-1)^L$. If spins are antiparallel ($S = 0$). spin wave function is antisymmetric, while for parallel spins ($S = 1$) it is symmetric – restoring initial spin configuration gives additional $(-1)^{S+1}$ factor

Parapositronium ($C = 1$) $\rightarrow \gamma + \gamma$

Ortopositronium ($C = -1$) $\rightarrow \gamma + \gamma + \gamma$

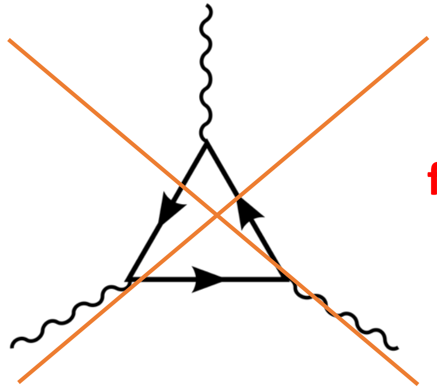


Symmetries in QED: examples II

- In the lowest order of perturbation theory Furry's theorem states that the probability is zero
- One can speculate that in any order there are such loops with odd number of photons and therefore the process has zero probability
- On the other hand, the process $\gamma + \gamma \rightarrow \gamma + \gamma + \gamma$ is forbidden in general, since

$$(-1)^{n_{\gamma,\mathrm{initial}}} \neq (-1)^{n_{\gamma,\mathrm{final}}}$$

just analyzing the initial and final states



forbidden

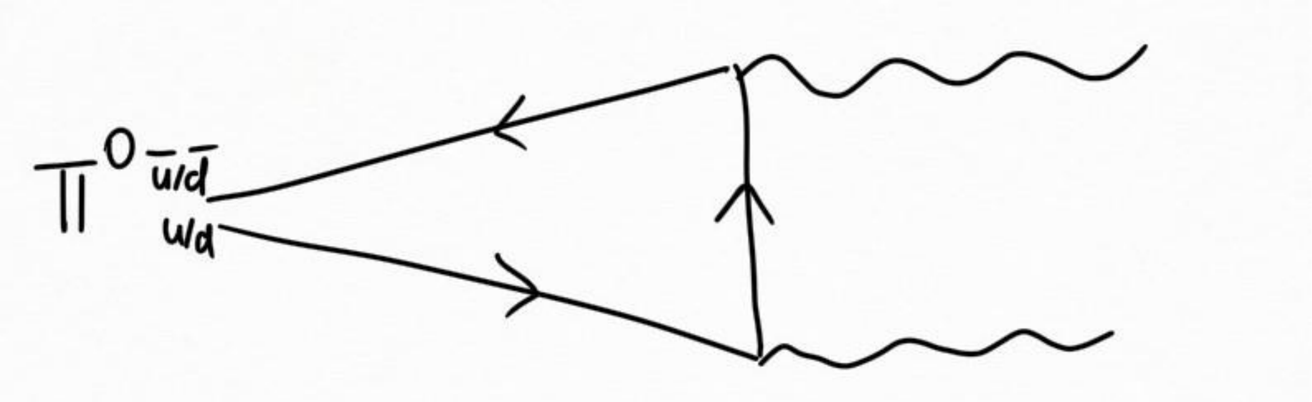
Symmetries in QED: examples II

- Neutral pion π^0 decays into two photons

$$\pi^0(u\bar{u} - d\bar{d}) \rightarrow \gamma + \gamma$$

- This is an electromagnetic annihilation process. From the final C −value, $C = (-1)^2 = 1$ we can infer that $C = 1$ for neutral pions

- Then a process $\pi^0 \rightarrow \gamma + \gamma + \gamma$ ($C_{\text{final}} = -1$) is forbidden!



Symmetries in general

- As we have shown, one can check if a process is allowed without even knowing explicitly physics beyond the process
- This happens because particles have quantum numbers related to symmetries
- If an interaction respects a symmetry, the corresponding quantum numbers must be conserved
  - a trivial example is conservation of electric charge
- Let's show how this works in general
- Assume we have a symmetry operator A that commutes with the Hamiltonian

$$[A, H] = 0$$

then the eigenstates of H are also eigenstates of A

Symmetries in general

- Consider a Hamiltonian that is a sum of a free particle Hamiltonian H_0 and interaction V: $H = H_0 + V$
- If A is a symmetry for both H_0 and V

$$[A, H_0] = [A, V] = 0$$

- then the eigenstates of H_0 (particles we observe) can be divided into sets with definite value of A. During the time evolution, the value of A does not change

Symmetries in general

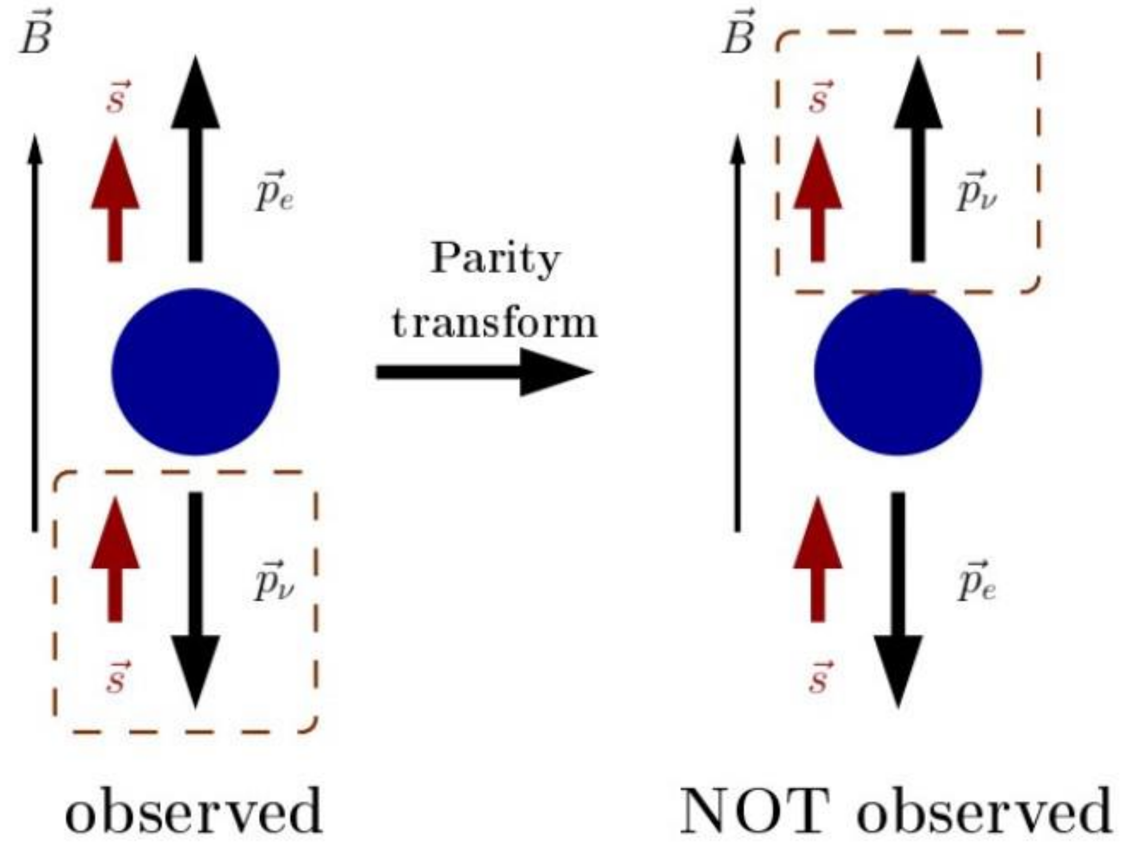
- If we start with an eigenstate of the initial Hamiltonian H_0, transitions are possible only into states with the same eigenvalues of A
- We can find out whether a process is forbidden by a symmetry by just analyzing initial and final states – **selection rules**
- The result does not depend on the explicit form of the interaction V, i.e., this works not only within the framework perturbation theory but in general

Examples of $\boldsymbol{P}$ violation: beta decay

Experimental setup

- ^{60}Co decays emitting two fermions: a neutrino and an electron
- During the transition the nuclei change their angular momentum by $\Delta J = 1$
- There are two options with different helicities
- They are related by parity but only one option is observed!

Examples of $\boldsymbol{P}$ violation: beta decay



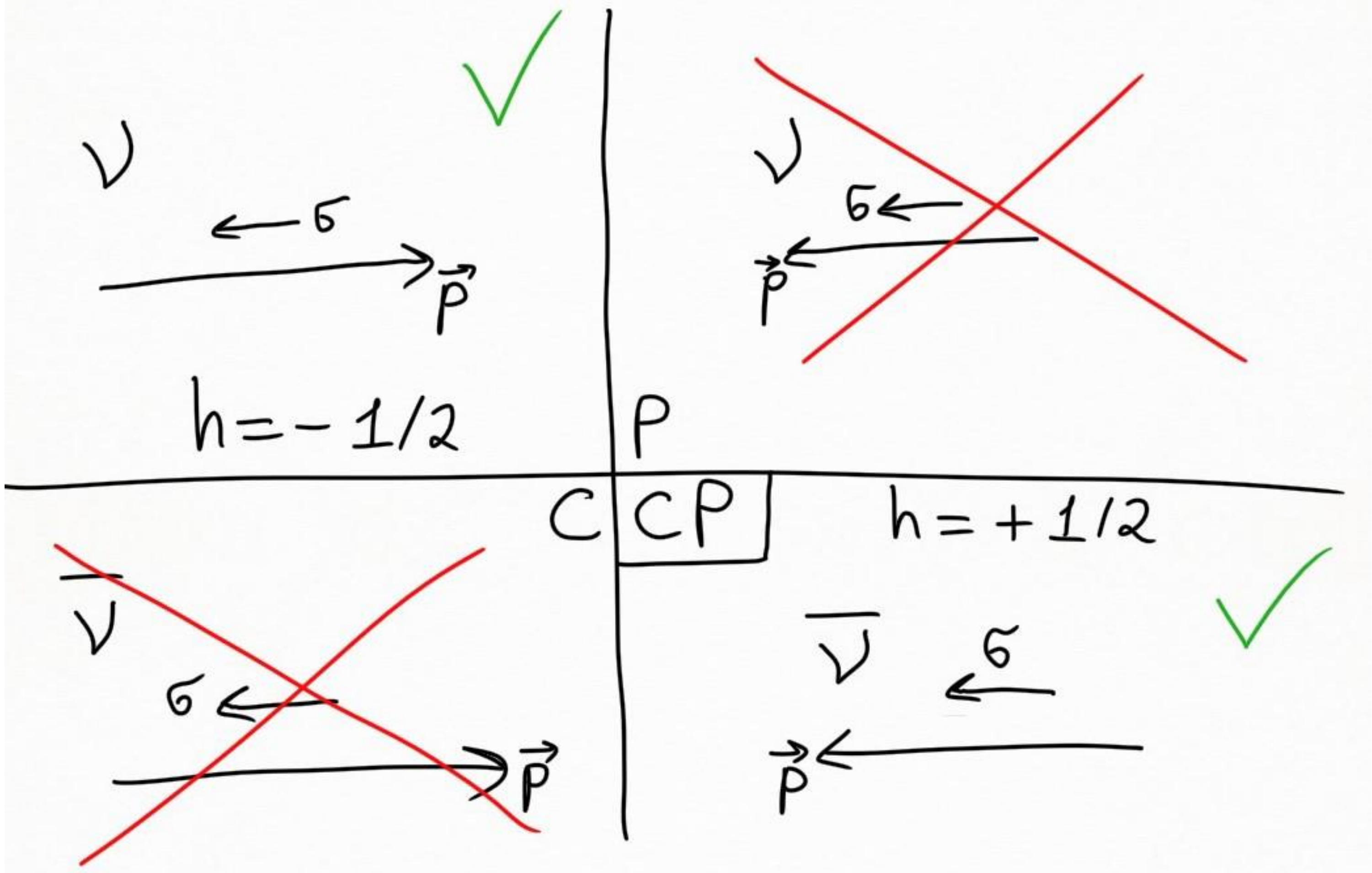
- In β decays of ^{60}Co only neutrinos whose spin is antiparallel to their momentum were observed

Parity is violated in β decays

Examples of $\boldsymbol{P}$ violation: beta decay

- Neutrinos are produced with positron and antineutrinos with electrons
- Antineutrinos produced in β −decays have only positive helicity (their spin is always parallel to their momentum)
- It seems that the actual symmetry of nature is the combined CP symmetry, when in addition to changing helicity we replace a particle by its antiparticle
- For fermions, parity transformation P could also include C conjugation. A particle looking in the mirror sees its antiparticle!

Examples of $\boldsymbol{P}$ violation: beta decay



- It seems that also CP is slightly violated in nature

Examples of $\boldsymbol{P}$ violation: beta decay

- To construct a theory of this process, we must use helicity states. However, the helicity states are not Lorentz-invariant!
- **Solution:** use eigenstates of γ_5 instead. In addition, γ_5 respects CP as well
  - this means that $\psi(\nu, h = -)$ and $\psi(\bar{\nu}, h = +)$ for massless neutrinos both belong to ψ_L spinor
- The Hamiltonian should contain only ψ_L
- Only left-handed neutrinos (with left chirality) interact weakly. If neutrinos were massless, the right-handed counterpart (right-handed neutrinos) would not interact with anything and therefore be redundant
- Although for electrons the difference between helicity and chirality plays some role (electrons are produced with both helicities), interactions at higher energies $E \gg m_e$ also reveal the fact that only the left component, ψ_L, interacts weakly

Kaon decays

- Neutral kaons differ from their antiparticles: $K^0(d\bar{s}) \neq \overline{K^0}(\bar{d}s)$
- These particles actually live in two superpositions: one with positive CP value $K_S(CP = 1)$ and one with negative one $K_L(CP = -1)$
- Pions have $P = -1$ and $C = 1$ (neutral). The allowed decays modes for the two neutral kaons are

$$K_S \to \pi\pi \qquad (\pi^0\pi^0 \text{ or } \pi^+\pi^-) \qquad CP = +1$$

$$K_L \to \pi\pi\pi \qquad CP = -1$$

(total angular momentum is zero)

- Since m_K only sightly exceeds $3m_\pi$, $K_L \to 3\pi$ decay width is kinematically suppressed and therefore $K_L(K - \text{long})$ lifetime is much larger than that of $K_S(K - \text{short})$

Summary

- We studied C and P symmetries
- Learned selection rules $-$ can be used to investigate if a process is forbidden by analysing initial and final states of an interaction (when the interaction and the free-particle Hamiltonian both conserve a given quantity)
- Next semester: unlike the strong and EM interactions, the weak interaction violates both P and CP symmetry
- One needs (beyond other prerequisites) both P and CP violation to accumulate charge

Summary of Lecture 13

Main learning outcomes

- Discussion on discrete symmetries: charge conjugation and parity
- Symmetry transformation for fermions
- Meson parity, helicity and chirality
- Examples of symmetry violation